Multi-phase flow modelling of transport, deposition and erosion of sediments

Suspended flow

The following system of equations describes multi-phase flow behaviour for sediment gravity flows. Generalized equations about suspension flow modelling may be found in Ishii (1975) and Ungarish (1993), and the suspension flow model implemented in MassFLOW-3D can be further read about in FLOW-3D (2009) and Basani & Hansen (2009). This system consists of seven groups of equations (1a, b)-(6) and seven groups of unknowns $\bar{\mathbf{u}}, P, c_{s,i}, \mathbf{u}_{r,i}, \mathbf{u}_{drift,i}, K_i, \mathbf{u}_{lift,i}$ (i = 1,N; N- total number of sediment species):

$$\nabla \cdot \bar{\mathbf{u}} = 0, \quad \frac{\partial \bar{\mathbf{u}}}{\partial t} + \bar{\mathbf{u}} \cdot \nabla \bar{\mathbf{u}} = -\frac{1}{\bar{\rho}} \nabla P + \mathbf{g} + \nabla\big((\nu + \nu_{\mathrm{T}}) \cdot \nabla \bar{\mathbf{u}}\big) \quad (1\mathrm{a}, \mathrm{b})$$

$$\left(\frac{\partial c_{s,i}}{\partial t}\right)_x + \bar{\mathbf{u}} \cdot \nabla c_{s,i} = D \nabla^2 c_{s,i} - \mathbf{u}_{lift,i} \cdot \nabla c_{s,i} - \mathbf{u}_{drift,i} \cdot \nabla c_{s,i} \quad (2)$$

$$\mathbf{u}_{r,i} = \frac{\mathbf{g}}{K_i}\left(\rho_{s,i} - \bar{\rho}\right) f_{s,i} \qquad (3)$$

$$\mathbf{u}_{drift,i} = \left(1 - f_{s,i}\right)\mathbf{u}_{r,i} - \sum_{j=1}^{N(-i)} f_{s,j}\, \mathbf{u}_{r,i} \qquad (4)$$

$$K_i = \frac{3}{4}\frac{f_{s,i}}{d_{s,i}}\left(C_{D,i}\left\|\mathbf{u}_{r,i}\right\| + 24\frac{\mu_f}{\rho_f d_{s,i}}\right) \qquad (5)$$

$$\mathbf{u}_{lift,i} = \alpha_i \mathbf{n}_s d_*^{0.3}\left(\theta_i - {\theta''}_{cr,i}\right)^{1.5}\sqrt{\frac{\|\mathbf{g}\| d_{s,i}\left(\rho_{s,i} - \rho_f\right)}{\rho_f}} \qquad (6)$$

1) a, b) $\bar{\mathbf{u}} \stackrel{\text{def}}{=} \left(1 - \sum_{j=1}^{N} f_{s,j}\right)\mathbf{u}_f + \sum_{j=1}^{N} f_{s,j}\, \mathbf{u}_{s,i}$ - the mean velocity of the fluid/sediment mixture (bulk flow), P –pressure
2) $c_{s,i}$ - the concentration of the suspended sediment, in units of mass per unit volume
3) $\mathbf{u}_{r,i} \stackrel{\text{def}}{=} \mathbf{u}_{s,i} - \mathbf{u}_f$ - the relative velocity
4) $\mathbf{u}_{drift,i} \stackrel{\text{def}}{=} \mathbf{u}_{s,i} - \bar{\mathbf{u}}$ - the drift velocity to compute the transport of sediment due to drift
5) K_i - drag function
6) $\mathbf{u}_{lift,i}$ - the entrainment lift velocity (volumetric flux) of sediment

Now we describe how this system of equations was derived and what it describes.

Equation (1a) is mass balance equation for incompressible fluid-sediment mixture.

Equation (1b) is Unsteady Reynolds-averaged Navier-Stokes equation for fluid-sediment mixture (bulk flow), where ν - molecular viscosity and ν_T - turbulent viscosity related to Reynolds stresses and $\nu_T \gg \nu$. Turbulent viscosity is calculated by RNG turbulence model (Lyn 2008). The RNG $k-\varepsilon$ model has several advantages over the standard $k-\varepsilon$ model. It is more accurate for rapidly strained flows and swirling flows and for lower Reynolds numbers (*Re*), the RNG model behaves better than the standard $k-\varepsilon$ which is only valid for high Reynolds number flows.

Short description of RNG model implemented in MassFLOW-3D™

The kinematic turbulent viscosity is computed from

$$\boldsymbol{\nu_T} = 0.085\frac{k_T^2}{\varepsilon_T}$$

The two-equations renormalization group (RNG) k-ε turbulence model is based on the turbulence viscosity hypothesis and solve two transport equations for the turbulent kinetic energy, k_T and the turbulent dissipation, ε_T.

The turbulent kinetic energy is defined as the energy of the turbulent velocity fluctuations:

$$k_T = \frac{1}{2}\left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2}\right)$$

where u', v' and w' are the component of the velocity fluctuations.

The transport equation for the turbulent kinetic energy k_T is

$$\frac{\partial k_T}{\partial t} + \left\{u\frac{\partial k_T}{\partial x} + v\frac{\partial k_T}{\partial y} + w\frac{\partial k_T}{\partial z}\right\} = P_T + G_T + D_K - \varepsilon_T$$

The transport equation of the turbulence dissipation ε_T is as follow:

$$\frac{\partial \varepsilon_T}{\partial t} + \left\{u\frac{\partial \varepsilon_T}{\partial x} + v\frac{\partial \varepsilon_T}{\partial y} + w\frac{\partial \varepsilon_T}{\partial z}\right\} = \frac{C_{\varepsilon 1}\cdot\varepsilon_T}{k_T}\left(P_T + C_{\varepsilon 3}\cdot G_T\right) + D_\varepsilon - C_{\varepsilon 2}\frac{{\varepsilon_T}^2}{k_T}$$

The rate of turbulent energy dissipation, ε_T, in one-equation model is related to the turbulent kinetic energy k_T:

$$\varepsilon_T = 0.085\frac{\sqrt{3/2}{k_T}^{3/2}}{TLEN}$$

The RNG model uses equations similar to the equations for the k-ε model. However, equation constants that are found empirically in the standard k-ε model are derived explicitly in the RNG model, like $C_{\varepsilon 2}$ is computed from the turbulent kinetic energy (k_T) and turbulent production (P_T) terms.

In order to prevent unphysical small dissipation rates, the minimum dissipation is limited by a maximum length scale, represented by TLEN in MassFLOW-3D™.

$$\varepsilon_{T,min} = 0.085\frac{\sqrt{3/2}{k_T}^{3/2}}{TLEN}$$

TLEN and the initiation of the turbulent kinetic energy k_T are the two main turbulent parameters that are defined by the user during the setting up of a model. *Equation (2)* is mass balance equation for each sediment species *i*, which describes the motion of the suspended sediment in the system by the advection-diffusion equation, with the addition of the effect of drifting and lifting of the sediment.

Equations (3) is obtained by subtracting momentum balance for fluid-sediment mixture (1) from momentum balances for each sediment species *i*, use of relative velocity definition and assumptions: a) that the motion of sediment is nearly steady at the scale of the computational time and that the advection term is very small due to small gradients in the drift velocity, b) that the ratio of pressure gradient to mixture density is typically proportional to the acceleration of gravity, g.

Equation (4) is derived from the definitions for $\bar{\mathbf{u}}$, $\mathbf{u}_{r,i}$ and $\mathbf{u}_{drift,i}$.

Equation (5) is obtained by combining of form drag and Stokes drag, (Clift et al. 1978).

Equation (6) is modelled to calculate the entrainment lift velocity (volumetric flux) of sediment and the entrainment coefficient α_i are based on Mastbergen and Van Den Berg (2003).

Bed-load transport and packing of sediments

Bed-load transport describes the movement of large particles along the surface of the bed without being entrained into the bulk fluid flow. Meyer-Peter & Muller (1948) formula predicts the volumetric flow rate of sediment per unit width over the surface of the packed bed.

The mass flux of sediment as computed across computational cell boundaries in MassFLOW-3D™ is a multiplication of velocity of the sediment in each computational cell, bed load thickness, the volume fraction of the bed-load layer and density of the sediment species,

$Q_{b,i} = \boldsymbol{u}_{bedload,i}\delta_i f_{b,i}\rho_{s,i}$Hindered settling is not important for the low concentration of the sediments.

Additional notations

ρ_f-fluid density

$\rho_{s,i}$ - density of the sediment species,

$\bar{\rho}$ - mean density

$f_{s,i}$- volume fraction of sediment species *i*

$d_{s,i}$ - diameter for sediment species i,

$C_{D,i}$ - drag coefficient for sediment species i,

P – pressure,

$P_T = C_{SP}\left(\frac{\mu}{\rho V_f}\right)\begin{bmatrix} 2\left(\frac{\partial u}{\partial x}\right)^2+2\left(\frac{\partial v}{\partial y}\right)^2+2\left(\frac{\partial w}{\partial z}\right)^2 \\ +\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)^2+\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)^2 \end{bmatrix}$- turbulent production term,

C_{SP} - turbulent parameter, whose default value is 1.0,

$G_T = -C_\rho\left(\frac{\mu}{\rho^3}\right)\left(\frac{\partial \rho}{\partial x}\frac{\partial p}{\partial x}+\frac{\partial \rho}{\partial y}\frac{\partial p}{\partial y}+\frac{\partial \rho}{\partial z}\frac{\partial p}{\partial z}\right)$- the buoyancy production term,

C_ρ- has a default value of 0.0 unless the problem is thermally buoyant, in which case it takes a value of 2.5,

$D_K = \frac{\partial}{\partial x}\left(\frac{v_T}{\sigma_k}\frac{\partial k_T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{v_T}{\sigma_k}\frac{\partial k_T}{\partial y}\right)+\frac{\partial}{\partial z}\left(\frac{v_T}{\sigma_k}\frac{\partial k_T}{\partial z}\right)$- diffusion term for the turbulent kinetic energy,

σ_k has avalue of 0.72 for the RNG model

$C_{\varepsilon 1}, C_{\varepsilon 2}$ *and* $C_{\varepsilon 3}$ - non-dimensional parameters. The default value for $C_{\varepsilon 1}$ is 1.42. $C_{\varepsilon 2}$ is computed based on the value of k_T, ε_T and the shear rate. $C_{\varepsilon 3}$ has a value of 0.2.

$D_\varepsilon = \frac{\partial}{\partial x}\left(\frac{v_T}{\sigma_\varepsilon}\frac{\partial \varepsilon_T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{v_T}{\sigma_\varepsilon}\frac{\partial \varepsilon_T}{\partial y}\right)+\frac{\partial}{\partial z}\left(\frac{v_T}{\sigma_\varepsilon}\frac{\partial \varepsilon_T}{\partial z}\right)$- diffusion term for the dissipation

where σ_ε is equal to 0.72 for the RNG model

$\mathbf{n}_s$ **–** outward pointing normal vector to the packed bed interface**,**

d_*- dimensionless mean particle diameter,

$||\mathbf{g}||$- magnitude of the gravitational vector,

μ_f - fluid viscosity

$\theta_i = \frac{\tau}{||\mathbf{g}|| d_{s,i}(\rho_{s,i}-\rho_f)}$- local Shields parameter at the bed interface (it is computed based on the local shear stress, τ)

$f_{b,i} = 0.18\frac{f_{packed}}{d_*}\left(\frac{\theta_i}{\theta''_{cr,i}} - 1\right)$- the volume fraction of the bed-load layer is predicted by *van Rijn, 1984,*

$\frac{\delta_i}{d_{50}} = 0.3d_*^{0.7}\left(\frac{\theta_i}{\theta''_{cr,i}} - 1\right)^{0.5}$ - bed load thickness, d_{50}- local mean particle size in the computational cell

$\boldsymbol{u}_{bedload,i} = u_{bedload,i}\frac{\overline{\boldsymbol{u}}}{\|\overline{\boldsymbol{u}}\|}$- the direction of the motion is determined from the motion of the liquid adjacent to the packed bed interface,

$u_{bedload,i} = \frac{q_{b,i}}{\delta_i f_{b,i}}$- velocity of the sediment in each computational cell,

$q_{b,i} = \beta_i\left(\theta_i - \theta''_{cr,i}\right)^{1.5} \cdot \left[\|\mathbf{g}\|\left(\frac{\rho_{s.i}-\rho_f}{\rho_f}\right)d_{s,i}^3\right]^{\frac{1}{2}}$- volumetric bed-load transport rate per unit width,

β_i- bedload coefficient is typically equal to 8.0 (Van Rijn 1984),

$\theta''_{cr,i} = \theta'_{cr,i}\frac{\cos\psi\sin\beta+\sqrt{\cos^2\beta\tan^2\phi_i+\sin^2\psi\sin^2\beta}}{\tan\phi_i}$- the critical Shields parameter modified for sloping surfaces to include the angle of repose (Soulsby 1997),

$\theta'_{cr,i} = \theta_{cr,i}\frac{1.666667}{\log_{10}\left(19\frac{d_{s,i}}{d_{50}}\right)^2}$- the critical Shields parameter modified by the effect of armoring (Egiazaroff 1965),

β- computed angle of the packed interface normal relative to the gravitational vector g,

ϕ_i, ψ- user-defined angle of repose for sediment species *i* and angle between the flow and the upslope direction, respectively

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